



1. Definition

A matrix is reduced row-echelon form if it satisfies the following requirements.

- (i) It is I row-echelon form
- (ii) The leading entry in each non zero row is I.
- (iii) All other elements of the column in which the leading entry I occurs are zeros.
- (iv)

$$\begin{aligned} 1.1. \quad & 3x - 2y + z = 0 \\ & -11x + 8y + z = 0 \\ & 10x - 7y + z = 0 \\ & -4x + 3y + z = 0 \end{aligned}$$

$$1.1.1. \quad \begin{bmatrix} 3 & -2 & 1 \\ -11 & 8 & 1 \\ 10 & -7 & 1 \\ -4 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\text{When } A = \begin{bmatrix} 3 & -2 & 1 \\ -11 & 8 & 1 \\ 10 & -7 & 1 \\ -4 & 3 & 1 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \underline{0} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$AX = \underline{0}$$

$$\begin{bmatrix} 3 & -2 & 1 \\ -11 & 8 & 1 \\ 10 & -7 & 1 \\ -4 & 3 & 1 \end{bmatrix}$$

Applying $R_4 \rightarrow R_4 + R_1$, $R_3 \rightarrow R_2 + R_3$

$$\begin{bmatrix} 3 & -2 & 1 \\ -11 & 8 & 1 \\ -1 & 1 & 2 \\ -1 & 1 & 2 \end{bmatrix}$$

Applying $R_4 \rightarrow R_4 - R_3$, $R_1 \rightarrow R_1 + 2R_4$, $R_3 \rightarrow R_3 + \frac{1}{3} R_1$ $R_2 \rightarrow R_2 - 11R_4$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & -3 & -21 \\ 0 & \frac{1}{3} & \frac{7}{3} \\ 0 & 0 & 0 \end{bmatrix}$$

Applying $R_2 \rightarrow -\frac{1}{3} R_2$, $R_3 \rightarrow 3R_3$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 7 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \end{bmatrix}$$

Applying $R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore \text{RREM (A)} = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$1.1.3. \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\therefore x + 5z = 0, y + 7z = 0$$

Take $z = \lambda$ as a parameters then the solution set
 $\{ (-5\lambda, -7\lambda, \lambda) \}$ (λ is a parameter)

$$1.2. \begin{aligned} x + 2y + z + t &= 1 \\ x + 3z - 2t &= 11 \\ x - y + z &= 8 \end{aligned}$$

Augmented matrix corresponding the above system of linear equations

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 3 & 0 & 3 & -2 & 11 \\ 1 & -1 & 1 & 0 & 8 \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 - 3R_1, R_3 \rightarrow R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 0 & -6 & 0 & -5 & 8 \\ 0 & 0 & 0 & -3 & -6 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 0 & -6 & 0 & -3 & 8 \\ 0 & -3 & 0 & -1 & 7 \end{bmatrix}$$

Applying

$R_3 \rightarrow 2R_3 - R_2$

$$R_3 \rightarrow \frac{-1}{3}R_3 \quad \begin{bmatrix} 1 & 2 & 1 & 1 & 1 \\ 0 & 6 & 0 & -5 & 8 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 \rightarrow R_1 - R_3, R_2 \rightarrow R_2 + 5R_3$

$$\begin{bmatrix} 1 & 2 & 1 & 0 & -1 \\ 0 & -6 & 0 & 0 & 18 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$$R_2 \rightarrow -\frac{1}{6}R_2$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

Applying $R_1 \rightarrow R_1 \rightarrow 2R_2$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 0 & 5 \\ 0 & 1 & 0 & 0 & -3 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

\therefore solution, $x + z = 5$

$y = -3$

$z = 2$

take $x = \lambda$, λ is a parameter $\{(\lambda - 3, 5 - \lambda, 2) : \lambda \text{ is free variable}\}$

2. Definition

An $m \times n$ matrix A is said to have a rank r if it has at least one submatrix of order r which is non-singular but all submatrices of order greater than r are singular.

2.1 Let $A = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 2 & 3 & 6 & 5 \\ 1 & -1 & -2 & 0 \end{bmatrix}$

The determinants of sub matrices of order 4 of A .

$$|A_1| = \begin{vmatrix} 2 & 4 & 3 \\ 3 & 6 & 5 \\ -1 & -2 & 0 \end{vmatrix} = 2 \begin{vmatrix} 2 & 2 & 3 \\ 3 & 3 & 5 \\ -1 & -1 & 0 \end{vmatrix} = 0$$

$$|A_2| = \begin{vmatrix} 1 & 4 & 3 \\ 2 & 6 & 5 \\ 1 & -2 & 0 \end{vmatrix} = 2 \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 1 & -1 & 0 \end{vmatrix}$$

$$\begin{aligned} C_2 &\rightarrow C_1 + C_2 \\ &= 2 \begin{vmatrix} 1 & 3 & 3 \\ 2 & 5 & 5 \\ 1 & 0 & 0 \end{vmatrix} = 0 \end{aligned}$$

$$C_2 \rightarrow C_1 + C_2$$

$$|A_3| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 3 & 5 \\ 1 & -1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & 3 & 3 \\ 2 & 5 & 5 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

$$|A_4| = \begin{vmatrix} 1 & 2 & 4 \\ 2 & 3 & 6 \\ 1 & -1 & -2 \end{vmatrix} = 2 \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 3 \\ 1 & -1 & -1 \end{vmatrix} = 0$$

\therefore The values of all the determinants of orders are zero and the sub matrix

$$A^1 \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \therefore |A^1| = -5 \neq 0$$

\therefore Rank $A = 2$

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 4 & 3 & 2 & 1 \\ 6 & 7 & 0 & 1 \\ 7 & 9 & -1 & 1 \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 - 4R_1$, $R_3 \rightarrow R_3 - 6R_1$, $R_4 \rightarrow R_4 - 7R_1$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -5 & 6 & 1 \\ 0 & -5 & 6 & 1 \end{bmatrix}$$

Applying $R_4 \rightarrow R_4 - R_2$, $R_3 \rightarrow R_3 - R_2$, $R_2 \rightarrow -\frac{1}{5}R_2$

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & -\frac{6}{5} & \frac{1}{5} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \rightarrow R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & \frac{7}{5} & \frac{2}{5} \\ 0 & 1 & -\frac{6}{5} & \frac{1}{5} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned}
2.3.1. \quad & x + 2y = \alpha \\
& 3x + y = \beta \\
& 2x - y = \gamma
\end{aligned}$$

The augmented matrix is

$$\begin{bmatrix} 1 & 2 & \alpha \\ 3 & 1 & \beta \\ 2 & -1 & \gamma \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 - 3R_1$, $R_3 \rightarrow R_3 - 2R_1$

$$\begin{bmatrix} 1 & 2 & \alpha \\ 0 & -5 & \beta - 3\alpha \\ 0 & -5 & \gamma - 2\alpha \end{bmatrix}$$

Applying $R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} 1 & 2 & \alpha \\ 0 & -5 & \beta - 3\alpha \\ 0 & 0 & \alpha + \gamma - \beta \end{bmatrix}$$

The system is consistent if and only if

$$\alpha + \gamma - \beta = 0 \quad \text{i. e. } \beta = \alpha + \gamma$$

If $\beta = \alpha + \gamma$

$$\begin{bmatrix} 1 & 2 & \alpha \\ 0 & -5 & \alpha + \gamma - 3\alpha \\ 0 & 0 & 0 \end{bmatrix} \quad \text{if } \beta \neq \alpha + \gamma. \quad \text{The system is inconsistent no solutions.}$$

$$R_2 \rightarrow -\frac{1}{5}R_2 \quad \begin{bmatrix} 1 & 2 & \alpha \\ 0 & 1 & \frac{-\gamma + 2\alpha}{5} \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 3 & -1 & 5 & 2 \\ 4 & 1 & \mu^2 - 14 & \mu + 2 \end{bmatrix}$$

Applying $R_2 \rightarrow R_2 - 3R_1$ and $R_3 \rightarrow R_3 - 4R_1$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & -7 & 14 & -10 \\ 0 & -7 & \mu^2 - 2 & \mu - 14 \end{bmatrix}$$

Applying $R_1 \rightarrow R_2 - 2R_2$, $R_3 \rightarrow R_3 - R_2$ and $R_2 \rightarrow -\frac{1}{7}R_2$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & \mu^2 - 16 & \mu - 4 \end{bmatrix}$$

Applying $R_1 \rightarrow R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 1 & \frac{8}{7} \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & \mu^2 - 16 & \mu - 4 \end{bmatrix}$$

Case one $\mu^2 - 16 \neq 0$ that is $\mu \neq \pm 4$ Applying $R_3 \rightarrow \frac{1}{\mu^2 - 16} R_3$

$$\begin{bmatrix} 1 & 0 & 1 & \frac{8}{7} \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & 1 & \frac{\mu - 4}{\mu^2 - 16} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & \frac{8}{7} \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & 1 & \frac{1}{\mu + 4} \end{bmatrix}$$

Applying $R_1 \rightarrow R_1 - R_3$ and $R_2 \rightarrow R_2 + 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 & \frac{8\mu + 25}{7(\mu + 4)} \\ 0 & 1 & 0 & \frac{10\mu + 54}{7(\mu + 4)} \\ 0 & 0 & 1 & \frac{1}{\mu + 4} \end{bmatrix}$$

If $\mu \neq 4$ the system has unique solution $x = \frac{8\mu + 25}{7(\mu + 4)}, y = \frac{10\mu + 54}{7(\mu + 4)}, z = \frac{1}{\mu + 4}$

Case $2\mu = -4$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & 0 & -8 \end{bmatrix} \text{ then the system is inconsistent no solution}$$

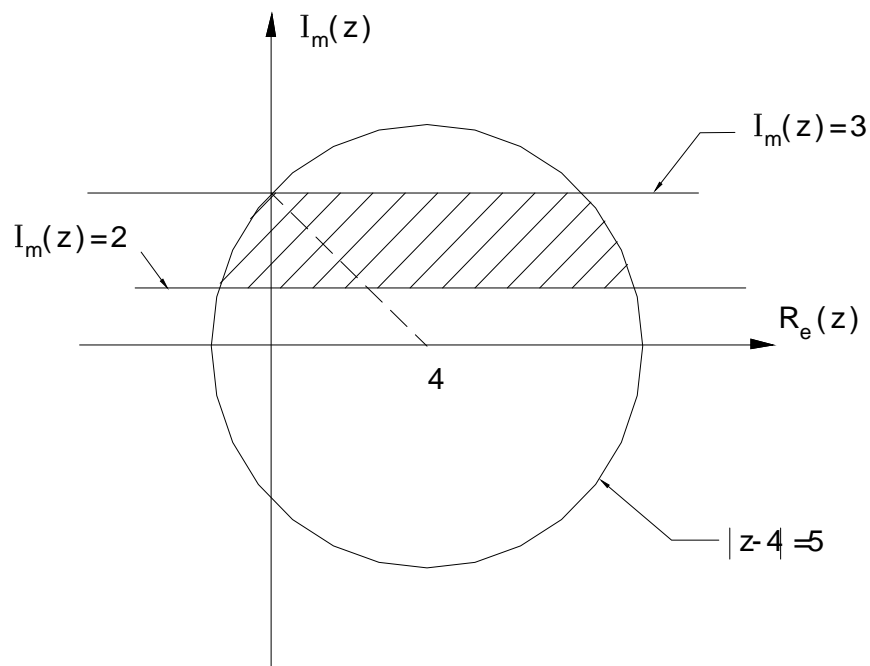
$$\mu = 4$$

$$\begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 1 & -2 & \frac{10}{7} \\ 0 & 0 & 0 & 0 \end{bmatrix} \text{ then the system is consistent the solution and infinitely many solutions}$$

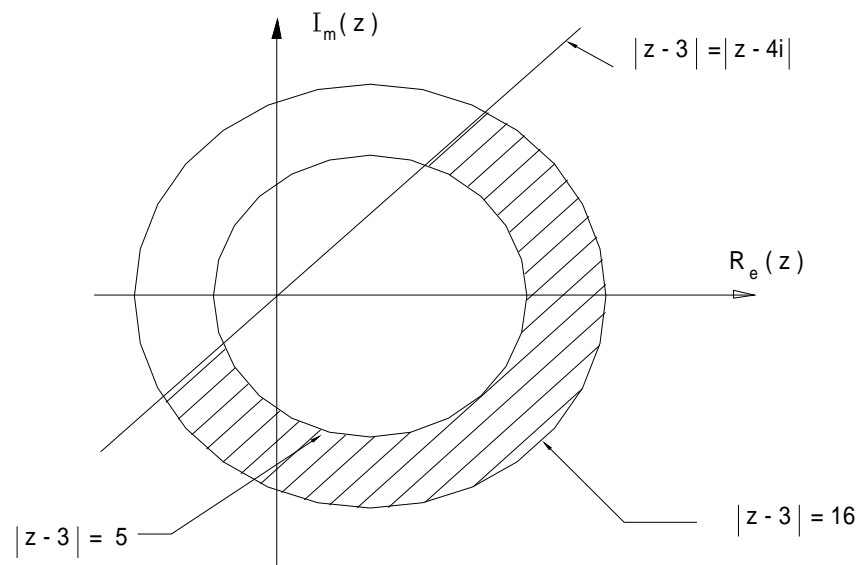
$$x = \frac{8}{7} - z, y = \frac{10}{7} + 2z \text{ and } z \text{ can take any value}$$

3.1

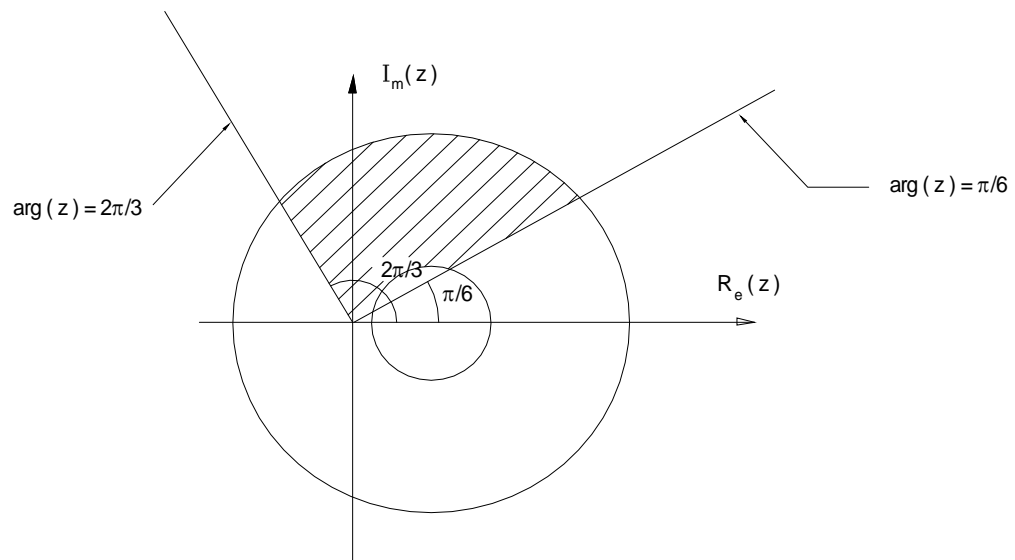
$$3.1.1. \quad |z-4| \leq 5 \text{ \& } 2 \leq I_m(z) \leq 3$$



3.1.2. $5 \leq |z - 3| \leq 16$ & $|z - 3| \leq |z - 4i|$



3.1.3.



3.2. Log (4 + 3i)

$$4 + 3i = 5 \left(\frac{4}{5} + i \frac{3}{5} \right)$$

$$\alpha = \tan^{-1} \left(\frac{3}{4} \right)$$

$$\begin{aligned} 4 + 3i &= 5(\cos \alpha + i \sin \alpha) \\ &= 5 \left[\cos(2h\pi + \alpha) + i \sin(2h\pi + \alpha) \right] \end{aligned}$$

$$4 + 3i = 5e^{(2h\pi + \alpha)i} (2h\pi + \alpha)i$$

3.2 Log (4 + 3i)

$$4 + 3i = 5 \left(\frac{4}{5} + i \frac{3}{5} \right)$$

$$\alpha = \tan^{-1} \left(\frac{3}{4} \right)$$

$$4 + 3i = 5(\cos \alpha + i \sin \alpha)$$

$$4 + 3i = 5 \left[\cos(2h\pi) + i \sin(2h\pi) \right]$$

$$4 + 3i = 5e^{(2h\pi + \alpha)i}$$

$$\begin{aligned} \text{Log}(4 + 3i) &= \text{Log} 5e^{(2h\pi + \alpha)i} \\ &= \ln 5 + (2h\pi + \alpha)i \end{aligned}$$

3.2.2. (1 + i)ⁱ

$$1 + i = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$= \sqrt{2} \left[\cos \left(2h\pi + \frac{\pi}{4} \right) + i \sin \left(2h\pi + \frac{\pi}{4} \right) \right]$$

$$1 + i = \sqrt{2} e^{\left(2h\pi + \frac{\pi}{4} \right)i}$$

$$\therefore \text{Log}(1 + i) = \ln \sqrt{2} + \left(2h\pi + \frac{\pi}{4} \right)i$$

$$i \log (1 + i) = i \ln \sqrt{2} - \left(2h\pi + \frac{\pi}{4} \right)$$

$$\text{Log}(1 + i)^I = i \ln \sqrt{2} - \left(2h\pi + \frac{\pi}{4} \right)$$

$$(1 + i)^i = e^{i \ln \sqrt{2} - 2h\pi - \pi/4}$$

$$= e^{i \ln \sqrt{2} - (2h+1)\pi/4}$$

3.3.

$$f(z) = \frac{3z+4}{z-1} \quad z \neq 1$$

Let , $z_1, z_2 \in D_f$

$$f(z_1) = f(z_2) \Leftrightarrow \frac{3z_1+4}{z_1-1} = \frac{3z_2+4}{z_2-1}$$

$$\Leftrightarrow 3z_1z_2 + 4z_2 - 3z_2 - 3z_1 - 4 = 3z_1z_2 + 4z_1 - 3z_2 - 4$$

$$\Leftrightarrow 7z_1 = 7z_2$$

$$\Leftrightarrow z_1 = z_2$$

$\therefore f(z)$ is one to one

Suppose $y = f(z) \Leftrightarrow z = f^{-1}(y) \quad z \neq 1$

$$y = \frac{3z+4}{z-1} \Leftrightarrow yz - y = 2z + 4$$

$$\Leftrightarrow (y-2)z = y+4$$

Clearly $y \neq 2 \quad z = \frac{y+4}{y-2}$

$$\therefore f^{-1}(y) = \frac{y+4}{y-2}$$

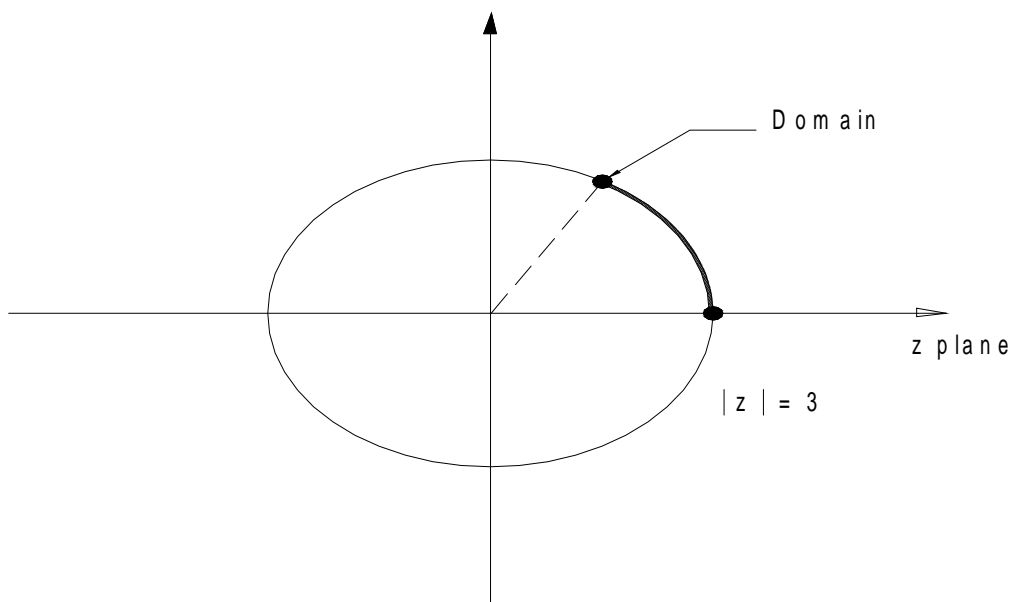
$$\therefore f^{-1}(z) = \frac{z+4}{z-2} \quad z \neq 2$$

3.4

$$f(z) = z^2, \quad |z| = 3$$

$$0 \leq \arg(z) \leq \frac{\pi}{3}$$

Domain $f(z)$



$$W = z^2$$

$$\arg w = \arg z^2 = 2\arg z$$

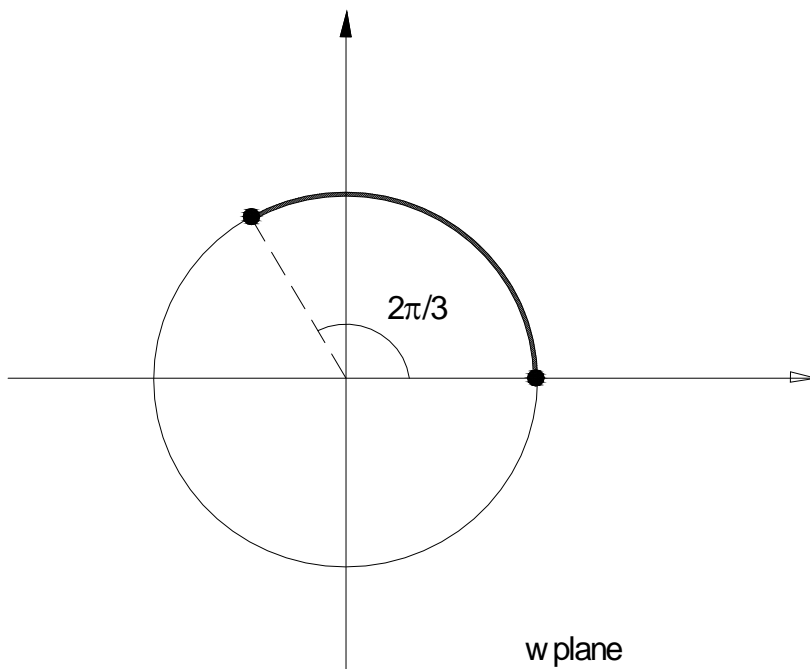
$$|w| = |z|^2 = 3^2$$

$$\text{Image } 0 \leq \arg z \leq \frac{\pi}{3}$$

$$0 \leq 2\arg(z) \leq \frac{2\pi}{3}$$

$$0 \leq \arg(w) \leq \frac{2\pi}{3}$$

$$|w| = 9$$



4.1

$$x \sim \text{Bin}(n, p)$$

$$\therefore M(t) = \sum_{x=0}^n n_{C_x} p^x q^{n-x} e^{t-x}$$

$$= \sum_{x=0}^n n_{C_x} (pe^t)^x q^{n-x}$$

$$= (pe^t + q)^n$$

$$\text{Since } p + q = 1$$

$$\begin{aligned}
M(t) &= (pe^t + 1-p)^n \\
M^1(t) &= n(pe^t + 1-p)^{n-1} pe^t \\
M^1(0) &= E(x) = n(pe^0 + 1-p)pe^0 \\
E(x) &= np \\
M^2(t) &= M^1(t) (pe^t + 1-p)^{n-1} + npe^t (pe^t + 1-p)^{n-2} (n-1)pe^t \\
M^2(c) &= E(x^2) = np(p + 1-p) + n(n-1)p^2(p + 1-p) \\
&= np + n(n-1)p^2 \\
V(x) &= E(x^2) - (E(x))^2 \\
&= np + n^2p^2 - np^2 - n^2p^2 \\
&= np(1-p)
\end{aligned}$$

n – number of toffees of a packet

4.2.

$$\begin{aligned}
np &= 11 \quad \text{----- (1)} \\
np(1-p) &= 4.95 \\
11(1-p) &= 4.95 \\
1-p &= 0.45 \\
P &= 1 - 0.45 \\
P &= 0.55 \\
n \cdot (0.55) &= 11 \\
n &= \frac{11}{0.55} = \frac{1100}{55} = 20 \\
n &= 20
\end{aligned}$$

4.2.2.

$$\begin{aligned}
p(X=x) &= {}^{20}C_x p^x (1-p)^{20-x} \\
p(x=8) &= {}^{20}C_8 (0.55)^8 (0.45)^{12}
\end{aligned}$$

4.2.3. Number of mango toffees in a packet $n = 100$

probability of a mango toffee

$$p = 0.3$$

probability of a another toffees

$$p = 1 - 0.3 = 0.7$$

$$\therefore X \sim \text{Bin} (n, p)$$

$$\text{Mean} = np = 100 \cdot 0.3 = 30$$

$$\text{Variance} = npq = 100 \cdot 0.3 \times 0.7 = 21$$

Using normal distribution

$$X \sim N(30, 21)$$

$$\therefore p(x < 3.5)$$

Using continuity connection

$$P(x < 34.5)$$

$$= p\left(\frac{x - \mu}{\sigma} < \frac{34.5 - \mu}{6}\right)$$

$$= p\left(z < \frac{34.5 - 30}{\sqrt{21}}\right) \quad z \sim N(0,1)$$

$$= p(z < 0.982)$$

$$= \Phi(0.982) \quad (\because p(z < a) = \Phi(a))$$

$$= 0.832$$

$$(x - \mu)^2 - 2\sigma^2 tx$$

$$= x^2 - 2(\mu + \sigma^2 t)x + \mu^2$$

$$= (x - \mu - \sigma^2 t)^2 + \mu^2 - (\mu + \sigma^2 t)^2$$

$$= (x - \mu - \sigma^2 t)^2 + \mu^2 - \mu^2 - 2\mu\sigma^2 t - \sigma^4 t^2$$

$$= (x - \mu - \sigma^2 t)^2 - (2\mu\sigma^2 t + \sigma^4 t^2)$$

$$M(t) = E(e^{tx})$$

$$= \int_{-\alpha}^{\alpha} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} e^{tx} dx$$

$$= \int_{-\alpha}^{\alpha} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2 + tx} dx$$

$$= \int_{-\alpha}^{\alpha} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}[(x-\mu)^2 - 2\sigma tx]} dx$$

$$= \int_{-\alpha}^{\alpha} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}(x-\mu-\sigma^2 t)^2} (2\mu\sigma^2 t + \sigma^4 t^2) dx$$

$$= e^{-\frac{1}{2\sigma^2}(-2\mu\sigma^2 t - \sigma^4 t^2)} \int_{-\alpha}^{\alpha} \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}(x-\mu-\sigma^2 t)^2} dx$$

$$M(t) = e^{-\mu t + \frac{\sigma^2 t^2}{2}}$$

$$M^1(t) = e^{\mu t + \frac{\sigma^2 t}{2}} \left(\mu + \frac{2\sigma^2 t}{2} \right)$$

$$M^1(0) = \mu \therefore E(x) = \mu$$

$$M^2(t) = e^{\mu t + \frac{\sigma^2 t}{2}} (\mu + \sigma^2 t)^2 + e^{\mu t + \frac{\sigma^2 t}{2}} (\sigma^2)$$

$$M^2(c) = \mu^2 + \sigma^2 \quad E(x^2)^2 = \mu^2 + \sigma^2$$

$$V(x) = E(x) - (E(x))^2 = \mu^2 + \sigma^2 - \mu^2 = \sigma^2$$

Suppose the random variable X is the time taken to germinate a one seed

Then $p(x > 6) = 0.2$ and $p(x < 4) = 0.1$

Taking μ and σ^2 as mean and variance of X $Z = \frac{x - \mu}{\sigma}$

Using the standard normal variable Z

$$P\left(Z > \frac{\sigma - \mu}{\sigma}\right) = 0.2 \quad \dots\dots\dots (1)$$

$$P\left(Z < \frac{4 - \mu}{\sigma}\right) = 0.1 \quad \dots\dots\dots [2]$$

$$\text{Using (1) } P\left(Z < \frac{\sigma - \mu}{\sigma}\right) = 0.8 \quad \dots\dots\dots [3]$$

Since $\frac{4 - \mu}{\sigma}$ is less than the mean value, so it is negative

From the symmetry of the z - curve

$$P\left(Z < \frac{4 - \mu}{\sigma}\right) = P\left(Z > \frac{\mu - 4}{\sigma}\right) = 0.1$$

$$P\left(Z < \frac{\mu - 4}{\sigma}\right) = 0.9 \quad \dots\dots\dots [4]$$

From the z - tables taking the nearest value of z giving these probabilities

$$\text{From equation [3] } \frac{6 - \mu}{\sigma} = 0.84 \quad \dots\dots\dots [5]$$

$$\text{From equation [4] } \frac{\mu - 4}{\sigma} = 1.28 \quad \dots\dots\dots [6]$$

From the equations [5] and [6] $\mu = 5.208$ and $\sigma = 0.943$

$\sigma = 6\text{cm}$ and $\mu = 140\text{ cm}$

Let x be the height of a tree then

$$P(x < 145) = P\left(\frac{X - 140}{6} < \frac{145 - 140}{6}\right) = P\left(Z < \frac{5}{6}\right)$$

$$P(x < 145) = P\left(Z < \frac{5}{6}\right) = P(Z < 0.833) = 0.7976$$

Let Y be the number of trees in the sample that is less than 145 cm tall. Then Y can take the value in the set {0, 1, 2, 3, 4, 5}

Clearly Y has a Binomial distribution with $n = 5$ and $p = 0.7976$

$$P(y = 5) = (0.7976)^5 = 0.323$$

$$P(y = 3) = {}^5C_3 (1 - 0.7976)^2 (0.7976)^3$$